Research Base of Effective Mathematics Instruction

McGraw-Hill's *Math Connects* Kindergarten through Algebra Series

Introduction

The National Council of Teachers of Mathematics (NCTM) first issued its standards for K-12 curriculum in 1989 (NCTM, 1989), standards for teaching in 1991 (NCTM, 1991) and standards for assessment in 1995 (NCTM, 1995). In 2000, they followed up with Principles and Standards for School Mathematics, an extensive update based on the latest research and thinking. Among its premises are that all students can learn mathematics; that specific instructional, assessment, and student-focused strategies are effective; and that certain types of professional development will likely improve students' mathematics achievement (NCTM, 2000). Most recently (NCTM, 2006), they released Curriculum Focal Points for PreKindergarten through Grade 8 Mathematics. This document was put forth as a beginning step for a coherent grade-by-grade curriculum.

The McGraw-Hill *Math Connects* series was designed to assist today's teachers with the challenge of helping students become mathematically proficient according to NCTM 2000 standards. In creating the new K-Algebra program, Glencoe and Macmillan/McGraw-Hill's goal was to reflect both the findings from key research on mathematics instruction, instructional best practices and curricuar focal points. This white paper documents the research findings to which the *Math Connects* series relates and provides examples of how the program practically applies that research.

A leading expert on the research foundations of the NCTM's 2000 standards initally recommended the basic set of research documents to review for this study. Additional research reviews, syntheses, and meta-analyses were identified based on follow-up searches of relevant databases.

According to the National Council of Teachers of Mathematics' (NCTM, 2000) Principles and Standards for School Mathematics, one of the most "robust" findings of educational and psychological research on learning complex subjects like mathematics is that "conceptual understanding is an important component of proficiency, along with factual knowledge and procedural facility" (NCTM, 2000, p. 20). This alliance, the text continues, "makes all three components usable in powerful ways" (NCTM, 2000, p. 20). For example, it helps students to be confident in determining when and how to use what they know; it makes subsequent mathematical learning easier; and it better prepares students to solve unfamiliar problems (NCTM, 2000).

Recent debates have ensued regarding mathematics research and the effect of the What Works Clearinghouse (WWC). Schoenfeld (2006a) questions the criteria for decisions of the WWC to use various math outcome measures as very restrictive. He suggests that the impact of math curricular evaluations are complex and adds that research in the 70s and 80s shift emphasis from student knowledge to enabling use of knowledge and process. The TIMSS (Beaton, Mullis, martin, Kelly and Smith, 1996) led the way to a more standards based curricula. The author further suggests that implementation of math curriculum may be answered by teachers appropriately trained. Rejoinders (Herman, Boruch, Powell, Fleischman and Maynard, 2006; Schoenfeld 2006b) further debate the issue.

The remainder of this paper is divided into seven major sections:
1. Curriculum Focal Points
2. Attaining Mathematical Proficiency
3. Ensuring Mathematical Proficiency To Inform Instruction
4. Assessing Mathematical Proficiency to Inform Instruction
5. Professional Development to Improve Mathematics Proficiency
6. Technology
7. Reading in the Content Areas

Each section presents a synthesis of relevant, scientifically-based research on effective K-Algebra mathematics teaching and learning. Also in each section, explaination is given as to how the *Math Connects* series is consistent with the research findings. The paper concludes with a comprehensive listing of research references.

Curriculum

The NCTM Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics

Over the years, there has been much debate as to the most salient mathematics topics to teach (NCTM, 2000). As noted elsewhere in the paper, accountability, especially the No Child Left Behind legislation, places teachers in the uncomfortable position of choosing from a long list of concepts, learning expectations and topics. To address the challenges presented, The National Council of Teachers of Mathematics (NCTM) recently published a document titled "Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence" (NCTM, 2006). The writing team of nine members consisted of a university professor and teachers representing the grades covered. The team utilized curricula from the United States and other countries as well as writings from math experts.

The document has as its focus a small number of what are called mathematical 'targets' (p 1) providing descriptions of the most salient math concepts and skills at each grade level. The document argues that organizing the curriculum around the focal points with an emphasis on process standards (NCTM, 1989) of communication, reasoning, representation connections and problem solving provides students with a coherent math body of knowledge.

How McGraw-Hill's Math Connects Series Relates to the NCTM Curriculum Focal Points

At each grade, McGraw-Hill's *Math Connects* series is closely related to the focal points suggested by NCTM. A few examples are presented here to demonstrate the point. At grade K, the focal points recommend representing, comparing and ordering whole numbers and joining and separating sets (Number and Operations), describing shapes and space (Geometry) and ordering objects by measurable attributes (Measurement). At Kindergarten, Chapter 2 lessons 2-1 to 2-6 presents the numbers 0 to 5, with lesson 2-7 comparing the numbers, and lesson 2-8 ordering the numbers. In Kindergarten chapter 3, shapes and patterns are introduced and compared and in chapter 7, the measurable attributes of length, height, weight and exploring capacity are covered.

At grade one, the focal points recommend developing understandings of addition and subtraction and strategies for basic addition facts and related subtraction facts (Number, Operations and Algebra), developing an understanding of whole number relationships, including grouping in tens and ones (Numbers and Operations) and composing and decomposing geometric shapes (Geometry). For first grade, in chapters 2 and 3, the book introduces addition/subtraction stories, modeling addition/subtraction and vertical addition/subtraction. In chapter 8, counting by tens, grouping by tens and skip counting is covered. Starting in chapter 12, solids, planes, faces and corners are presented; including positioning and problem solving.

In grade five, the recommended focal points include developing an understanding of and fluency with division of whole numbers (Number, Operations and Algebra), developing an understanding of and fluency with addition and subtraction of fractions and decimals (Numbers and Operations) and describing three dimensional shapes and analyzing their properties, including volume and surface area (Geometry, Measurement and Algebra). For grade 5, chapters 3 and 4 introduce decimals and fractions and chapter 5 begins problems of adding and subtracting fractions and decimals. In chapter 10, angles, quadrilaterals and two dimensional figures are introduced and chapter 11 follows with surface area, three dimensional objects and prisms.

As a final example, in grade 8, NCTM foci are analyzing and representing linear functions and solving linear equations and systems of linear equations (Algebra), analyzing two and three dimensional space and figures by using distance and angle (Geometry and Measurement) and analyzing and summarizing data sets (Data Analysis, Numbers and Operations and Algebra). *Glencoe Algebra* begins variable and expressions in chapter 1 followed by writing and solving equations in chapter 2. Geometry begins in chapter 4 with distance and angles covered in chapter 10. Data analysis via graphing starts in chapter 3, basic statistics in chapter 4 with statistics and probability in chapter 12.

The above are but four examples. An analysis grade by grade shows that McGraw-Hill's *Math Connects* series is extremely consistent with the NCTM focal points.

Attaining Mathematical Proficiency

<u>The Five Strands of Mathematical Proficiency</u>

Building on this premise for attaining mathematical proficiency, the Mathematics Learning Study Committee of the National Research Council's (NRC, 2001) Center for Education conducted a review and synthesis of relevant research on mathematics learning from pre-kindergarten through grade 8. From their review, the committee identified five critical "interwoven and interdependent . . . strandsmathematical proficiency":

1. conceptual understanding: comprehension of mathematical concepts, operations, and relations;
2. procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
3. strategic competence: ability to formulate, represent, and solve mathematical problems;
4. adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;

5. productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efforts (National Research Council, 2001, p. 5). Each of the five strands "supports and reinforces the others," noted the committee, so care should be taken to develop each of them concurrently (National Research Council, 2001, p. 409). Furthermore, "Textbooks and other instructional material should develop the core content of school mathematics in a focused way, in depth, and with continuity in and across all grades, supporting all strands of mathematical proficiency" (National Research Council, 2001, p. 421).

Evidence of the interconnectedness of these five strands of mathematical proficiency can be found throughout the research on mathematical learning in the elementary and middle school grades. For example, studies of successful instructional techniques in high-poverty classrooms found that striking a balance between teaching for conceptual understanding and teaching for procedural fluency contributed to the development of both fluency and problem-solving skills in high- and low-achieving students (Knapp, 1995, Zucker, 1995, as cited by Fuson, 2003). Summarizing the results of these studies, Fuson reported that teachers who were successful in eliciting higher performance in their students were those who focused on alternative methods to finding solutions, used "multiple representations and real-life situations to facilitate meaning-making," modeled skills that could uncover "the meaning of mathematical problems or methods," and were "supplied a 'healthy dose' of skills practice (Fuson, 2003, p. 89). Similarly, a comparative study of seventh graders in two mathematics classes concluded that such a balance matched well to students' effective use of metacognitive strategies (Lester, Garofalo, and Kroll, 1989, as cited by Schoenfeld, 1992).

Furthermore, numerous research studies on elementary school children's learning of arithmetic have found that "instructional programs that emphasize conceptual development, with the goal of developing students' understanding, can facilitate significant mathematics learning without sacrificing skill proficiency" (Carpenter, Fennema, Peterson, Chiang, and Loef, 1989, Cobb et al. 1991, Cognition and Technology Group at Vanderbilt, 1997, Hiebert and Wearne, 1993, 1996, Hiebert et al. 1997, Kamii and Joseph, 1989, Knapp, Shields, and Turnball, 1992, Mack, 1990, Wearne and Hiebert, 1988, Wood and Sellers, 1996, as cited by Hiebert, 1999, p. 14).

In addition, research suggests that "students can learn new concepts and skills while they are solving problems" (Hiebert, 1999, p. 14). Studies on alternative ways of teaching elementary mathematics have shown that conceptual understanding can be enhanced when teachers treat the development of a procedural skill, itself, as a problem for students to solve (Hiebert et al. 1996, as cited by Hiebert, 1999, pp. 14-15, Grevemeijer and van Galen, 2003).

Conceptual Understanding and Procedural Proficiency

Research from the cognitive sciences has found that the order in which one teaches concepts and procedures is critical to students' success with mathematics.

- Instruction designed to provide conceptual understanding should come first.
- Learning about and practicing procedures should come second for students.

A review of cognitive-science studies has found that focusing on procedural fluency before teaching for conceptual understanding puts students at a disadvantage for developing their conceptual understanding and problem-solving skills in the areas of multidigit addition and subtraction, for instance (Fuson and Briars, 1990, Hiebert and Wearne, 1996, as cited by Siegler, 2003). In exploring why, related studies have suggested that by memorizing and practicing procedures intensively, students have difficulty going back later and understanding them conceptually (Brownell and Chazal, 1935, Mack, 1990, Resnick and Omanson, 1987, Wearne and Hiebert, 1988, as cited by Hiebert, 1999).

Additional support for these findings come from a comparative study of six classes of fifth-graders that was designed to explore the effects from the sequence of instruction on learning of area and perimeter of squares, rectangles, triangles and parallelograms (Pesek and Kirshner, 2000). The researchers found in a qualitative analysis of interviews with students that cognitive, metacognitive, and attitudinal interferences all interrupted their ability to understand concepts for students who were taught procedures before receiving instruction focusing on conceptual understanding. Students exposed to only conceptual instruction demonstrated superior performance on a posttest requiring application of both procedural skill and conceptual understanding, compared to students who received procedural-before-conceptual instruction. This learning advantage approached statistical significance. (Pesek and Kirshner, 2000).

Another comparative study, involving fifth graders, found that students who had received conceptual instruction prior to procedural instruction made significantly greater gains in both areas than students who received procedural instruction first as well as students who didn't receive either type of instruction (Rittle-Johnson and Alibali, 1999, as cited by Siegler, 2003).

In a report on a study exploring how students learn whole-number mathematics, Fuson (2003) explained why the balanced approach is so effective in improving students' mathematical proficiency: Students can learn to understand and explain computational methods if these methods are approached as sense-making endeavors. Educators can call on problem solving from the beginning to give meaning to computations; then they can continually intertwine the two as methods and problem solving become consolidated. Research has also explored how to utilize students' prior, informal, mathematical understanding to good advantage. For example, a study of fifth graders learning to multiply and divide fractions found that encouraging students to frequently return to their initial conceptions was a viable way for students to develop understanding of complex content domains (Mack, 2001). Other studies of elementary mathematics instruction have determined that the use of visual supports, such as drawings and/or manipulatives, is an effective way to develop students' understanding of mathematical concepts (Mack, 2001; Beishuizen, Gravemeijer, and van Lieshout, 1997, Carpenter, Franke, Jacobs, and Fennema, 1998, Fuson and Briars, 1990, Fuson et al. 1997, as cited by Fuson, 2003).

A review and synthesis of 30 years of research on computational methods showed that learners "move through progressions of methods from initial, transparent, problem-modeling, concretely represented methods to less-transparent, more problem-independent, mathematically sophisticated, symbolic methods" (Fuson, 2003, p. 71). To assist students in making this gradual progression to mastery of "well-structured skills" such as basic mathematical algorithms, Rosenshine and Stevens reviewed correlational and experimental studies on teacher effects conducted from 1955 to 1980 (Rosenshine and Stevens, 1986, as cited by Rosenshine, 1995, p.264). They concluded that teachers should follow this instructional sequence:

- Start with a "short review of previous learning";
- Summarize the learning objectives for students;
- Present step-by-step instruction, with interspersed practice;
- Offer "clear and detailed…explanations";
- Provide all students with lots of "active practice": first teacher-guided practice, then later on, independent practice;
- Ask many questions and obtain answers from all students; and

•Provide systematic feedback and corrections to students in response to their work (Rosenshine and Stevens, 1986, as cited by Rosenshine, 1995, p.264).

Strategic Competency through Problem-Solving Skills

Research points to several specific strategies as being effective in helping students become mathematical problem solvers. A meta-analysis of 487 research reports on four areas of problem solving found that training students in the subskills of selecting the correct operations, translating verbal statements to mathematics, and using diagrams improved their problem-solving performance. Furthermore, reflecting how important representing the problem is as the initial activity toward a problem's successful solution, the meta-analysis showed that training students in diagram drawing delivered the largest performance improvement, and second was providing physical objects for them to use in modeling (Hembree, 1992).

To become good problem solvers, it is important for students to acquire "a mathematical point of view," noted Schoenfeld in his synthesis of research on cognition and mathematical proficiency (1992, p. 344). This can be achieved by creating a community of mathematical practice in the classroom, characterized by collaboration and discussion (Schoenfeld, 1992). For example, discussing solutions to routine problems can provide students with "learnable and usable" problem-solving strategies (Schoenfeld, 1992, p. 354). Based on their review of research on instruction in mathematical problem-solving, Heller and Hungate (1985, as cited by Schoenfeld, 1992) concluded that discussions about problem-solving should

• Make tacit processes explicit;
• Involve students in talking about processes;
• Provide a balanced focus on both qualitative understanding and knowledge of specific procedures, including their component procedures; and
• Be followed by guided practice (Heller and Huntgate, 1985, as cited by Schoenfeld, 1992).

Schoenfeld (1992) also concluded that continually examining students' progress is another key to success in mathematical problem solving. He found that coaching, guided practice, and active interventions by teachers helped students learn to self-monitor their own problem-solving processes and to more quickly abandon paths that were unlikely to lead to a correct solution.

Mathematical problem-solving tasks are inherently less structured than algorithmic procedures. Research on less structured learning tasks (Collins, Brown, and Newman, 1990, Pressley et al. 1990, and Rosenshine and Meister, 1994, as cited by Rosenshine, 1995) suggests that cognitive strategies—guiding strategies that support students as they develop the ability to solve less structured problems—can be effective if teachers:

• "Provide procedural prompts";
• "Teach the cognitive strategies using small steps";
• "Model the process of using the cognitive strategy";
• "Guide student practice" in using the cognitive strategy (Rosenshine, 1995, p. 266-267).

Finally, a recent review of mathematics-education research from a cognitive-science perspective found that students who used a greater variety of strategies for solving problems also tended to learn better subsequently (Alibali and Goldin-Meadow, 1993, Chi, de Leeuw, Chiu, and LaVancher, 1994, Siegler, 1995, as cited by Siegler, 2003). This occurs, in part, noted Siegler (2003), because the greater variety enables students to cope with whatever kinds of problems they encounter.

Adaptive Reasoning

Adaptive reasoning refers to one's "ability to think logically about the relationships among concepts and situations," explain the authors of Adding It Up (National Research Council, 2001, p. 129). In mathematics, they noted, "adaptive reasoning is the glue that holds everything together," used by students "to navigate through the many facts, procedures, concepts, and solution methods and to see that they all fit together in some way" (p. 129). Adaptive reasoning, for example, is how students determine if a particular solution strategy will be effective.
Hembree's (1992) meta-analysis of 487 studies found that, in this order, students' skill with analogies, reasoning ability, inferences, and critical thinking most strongly correlated to high problem-solving performance in mathematics. Other research noted that analogical reasoning, metaphors, and similar mental methods served students by first helping them understand mathematical problems and then as sources of problem-solving techniques (English, 1997, as cited by National Research Council, 2001).

Productive Disposition

Integral to the NCTM's vision of math instruction is that all K-8 students can be mathematically proficient (National Research Council, 2001). Teachers and students both have to truly believe it; a classroom's activities and atmosphere have to exude it. Fortunately, research has identified what motivates students to learn: they have to have the expectation that they can learn, and they have to appreciate that what they are learning is valuable (Feather, 1982, as cited by NRC, 2001). It is possible to turn this positive premise into practice. Research has thus determined that teachers will motivate students by:

- Designing tasks and goals to be achievable with a reasonable investment of effort;
- Providing appropriate scaffolding (e.g., physical manipulatives, visuals, dialogue) along the way; and,
- Highlighting why and how students will find the learning valuable (National Research Council, 1999, as cited by National Research Council, 2001).

A "productive disposition towards mathematics is a major factor" in determining students' success, concluded the authors of Adding It Up (NRC, 2001, p. 131).

As noted by the Mathematics Learning Study Committee, "The role of practice in mathematics is to be able to execute procedures automatically, without conscious thought—to achieve automaticity" (National Research Council, 2001, p. 351). Practice doesn't have to be disjointed, noted the committee. Students can easily be reinforcing previous skills within the context of solving new problems during practice (National Research Council, 2001). Practice improves a student's speed and accuracy of performance on cognitive tasks, research on expert performance has shown, when certain conditions are met:

- The student must be motivated to attend to the task;
- The task must be deliberately designed to build on the learner's pre-existing knowledge; and,
- The learner must receive immediate, informative feedback on their performance (Welford, 1968, as cited by Ericsson, Krampe, and Tesch-Römer, 1993).

Cognitive science research also suggests that distributing practice sessions over time leads to improved memory retention of facts (Donovan and Radosevich, 1999, as cited by Willingham, 2002). Furthermore, the longer the time between practice sessions, the fewer practice sessions are needed to achieve retention (Bahrick and Phelps, 1987, as cited by Willingham, 2002). Ericsson et al. (1993) concluded that spacing apart sessions (typically one session per day) of deliberate practice of behaviors specifically "designed to improve current level of performance" allows for assimilation and avoids "motivational burnout" (pp. 368, 371). Children need "scaffolded practice" to become more fluent in orchestrating the steps in any algorithm, noted a review of the research on whole-numbers learning (Fuson, 2003, p. 72). Similarly, Rosenshine's (1995) summary of research on teacher effects deemed guided practice an essential transitional step between demonstration and independent practice. His other conclusions concerning guided practice were as follows:

- All students need to be involved in extensive guided practice, and it should continue until students reach a high level of success.
- Guided practice should start with teacher modeling and include a variety of techniques, including problems worked out in front of the class, checking for understanding, whole-class discussion, and pair or small-group work.
- Teachers should provide frequent, immediate feedback during guided practice (typically after each problem is completed) to limit the development of misconceptions. Differentiated feedback for different categories of student responses should also be provided.
- Independent practice should occur after sufficient guided practice so that students do not end up practicing errors and misconceptions.
- Guided practice is advantageous in helping students to develop cognitive strategies that effectively apply to mathematical problem-solving tasks (Rosenshine, 1995).

Measurement

The measurement of math proficiency is inded complex. Smith (2005) notes that while there is federal funding for assessment , individual states still have the main responsibility for measuring student achievment. Smith identifies numerous obstacles to measuring student success at meeting math standards including student mobility (school to school, district to district, state to state) and the reliance on measures that fail to assess what students can do versus how the do nomatively. He advocates for meaningful and somewhat standardized montiorng intruments for the filed of mathematics.

Fletcher (2005) is consistent with the above by addressing the issue of measuring numbers, letters and words as compared to cognitive processing. The concern reflects both general measurement as well as screening for math disabilities.

Cognitive Instruction

Naylor (2006a, b, c, d, e, and f) provides numerous interactive ideas and activities to gain student interest and assist in instruction. The goals of these activities include promoting conceptual development, challenging students, identifying patterns, sharpening visualization of logical relationships and assisting with numerical and spatial awareness.

How McGraw-Hill's Math Connects Series Reflects the Research on Attaining Mathematical Proficiency

Macmillan/McGraw-Hill Math provides ample opportunity for students to develop the five strands critical for mathematical proficiency as identified by the research:

- Conceptual understanding
- Procedural fluency
- Strategic competence
- Adaptive reasoning
- Productive disposition

McGraw-Hill's *Math Connects* series develops students' conceptual understanding of core mathematical ideas primarily through hands-on experience involving visual supports and models, and related discourse. The program is rooted in the principle that conceptual understanding is of paramount importance. Daily instruction in the primary grades provides teachers with a related activity in the Teacher's Edition for every lesson taught. For example, specific manipulatives provide a concrete experience and discourse whenever a new concept is introduced. These engaging tasks are implemented before any attempt to address procedural fluency—as recommended by the research.

As recommended by the research, McGraw-Hill's *Math Connects* series takes a balanced approach to addressing the five strands. There is a logical sequence of instruction, typically starting with the conceptual, making the link to procedural algorithms and focusing on procedural fluency, then applying the concepts and procedures in problem-solving situations. Along the way, there are opportunities for students to reflect, explain, and justify their thinking in discussion and in writing, and to apply what they are learning to real-world scenarios, which help students come to view mathematics as making sense and having real value.

For all chapters at all levels, the strands of mathematical proficiency are developed concurrently by McGraw-Hill's *Math Connects* series. While there is an appropriate balance among the strands, conceptual development precedes the focus on procedural proficiency. The series demonstrates these research-based characteristics with respect to practice and feedback:

- Guided practice starts with teacher modeling and includes a variety of techniques.
- Guided practice is provided as an essential transitional step between demonstration and independent practice.
- Practice is distributed over time.
- Student understanding is checked frequently to limit the development of misconceptions.
- Ample background on math concepts and procedures is included in the Teacher's Edition,

Another of McGraw-Hill's *Math Connects* series guiding principles is that sufficient practice opportunities should be provided to enable students to master essential procedural algorithms and problem solving strategies, but students should not be overwhelmed with too much mindless and counterproductive drill.

Two illustrative examples are given here. At Grade 1, Chapter 12, lesson 12.5 covers problem solving strategies. The Teacher's Edition mirrors the Student Edition with assistance to teach the lesson effectively. A problem to be solved is introduced. In this case – Together Todd and Tracy have 11 carrots in their lunch. Todd has 6 carrots. How many does Tracy have? The Teacher's Editionoffers activity choices for guided practice with feedback. After the introduction of the problem, students are provided a structure to Understand (What do I know? What do I need to find out?), Plan (How will I solve the problem?), Solve and Check. The teacher then assigns additional activities and assesses. A second example is found in the Algebra text, Chapter 3, lesson 3.1, Functions and Patterns. At this level, the steps for the teacher are to Focus the student, Teach, then Assess. The activity comes from baseball with a matrix provided (ordered pairs) considering years, strikeouts and homeruns of a particular player. Again, a variety of activities are available for practice. A Teacher to Teacher section presnts an idea of pairing students in the class with their favorite color.

McGraw-Hill's *Math Connects* series logically interweaves the five critical strands of mathematical proficiency identified by the Mathematics Learning Study Committee of the NRC's Center for Education, and strikes an appropriate balance among the strands. However, also recommended by the research, introduction of key math concepts consistently precedes a focus on procedural fluency.

The program adheres to specific research-based strategies for developing conceptual understanding, procedural fluency, and strategic competence. Problem solving lessons provide the experience students need to develop adaptive reasoning skills and a productive disposition towards mathematics. The program provides ample opportunities for meaningful practice and teacher feedback, and it provides teachers with sufficient background to give students meaningful feedback.

Ensuring Mathematical Proficiency for All Learners

Intervention

Before examining the results of studies themselves, it is useful to to note that in the research regarding math intervention. Seethaler and Fuchs (2005) analyze the literature in terms of the efficacy of studies completed. They found that randomized, controlled designs were clearly underrepresented in the literature. The conclude that to truly assess efficacy, study methodology may need to improve. In a related article, Augustyniak, Murphy and Phillips (2005) argue that the research on the definition of a math disability is lacking with respect to identification of core deficits. They identify the core areas needing further explanation as numerical skills, visual/spatial deficits, cognitive skill development (memory retrieval, working memory, speed of processing, attention regulation, problem solving) and social cognition. Mazzocco (2005) reviewed research regarding practices of early identification and intervention for students with math diffciulties. The commentary discusses the criteria and nature of math difficulties and notes the need for additional research.

The above being said, Butler, Beckingham and Lauscher (2005) report on three case studies regarding the support of students with math learning challenges. Three eighth grade students were given assistance in self-regulating their learning. General strategies found to be successful included:

- engaging the students in constructive conversation;
- supporting students reflection on their learning; and,
- the need for teachers to engage in dynamic, curriculum based forms of assessment

Fuchs, Fuchs and Hamlett (2006) report on the validation of an intervention to improve math problems solving at the third grade. The intervention (Hot Math) involved explicit instructions, self-regulation strategies and tutoring. Results indicated positive, short term results for problem solving skills.

Stinson (2006) suggested that a focus on the discourse of achievment in mathematics rather than the discourses of deficiency and rejection could prove beneficial in reducing the well known achievment gap between white and Black students. He suggest that the limited amount of research shows that enrichment activities, mentoring competent teachers and Black students identifying with the 'good kids group' (p. 496) may enhance math achievement in Afican American males.

Research also suggests a variety of instructional strategies that are effective to meet the needs of students with special needs—including those with physical disabilities, mental impairments, and/or learning disabilities; English Language Learners (ELL); and low-performing students who require some special attention to bring out the best of their capabilities. Effective instruction for special-needs students, the research has found, includes:

- Setting clear goals for students (Bray and Turner, 1986, Cherkes-Julkowski and Gertner, 1989, Ferritti, 1989, Ferritti and Cavalier 1991, as cited by Baroody, 1996; Schunk, 1985, as cited by Mastropieri, Scraggs, and Shinh, 1991);
- Using a "big ideas" structure for concepts (Kameenui and Carnine, 1998, as cited by Fuson, 2003, p. 88);
- Teaching content that is not too difficult (Bray and Turner, 1986, Cherkes-Julkowski and Gertner, 1989, Ferritti, 1989, Ferritti and Cavalier 1991, as cited by Baroody, 1996; Baroody, 1996) and presented within meaningful contexts (Miller and Mercer, 1997, as cited by Allsopp, Lovin, Green, and Savage-Davis, 2003);
- Laying ample groundwork by providing background knowledge (Bray and Turner, 1986, Cherkes-Julkowski and Gertner, 1989, Ferritti, 1989, Ferritti and Cavalier 1991, as cited by Baroody, 1996; Kameenui and Carnine, 1998, as cited by Fuson, 2003);
- Modeling by teachers (Allsopp et al., 2003; Baroody, 1996; Blankenship, 1978, as cited by Mastropieri et al., 1991);
- Sequencing instruction to go from the concrete to the abstract (Miller and Mercer, 1997, as cited by Allsopp et al., 2003);
- Using mediated scaffolding (e.g., visual supports with cues, teachers' feedback on thinking, peer tutoring) (Kameenui and Carnine, 1998, as cited by Fuson, 2003);
- Discussing mathematics using language (Miller and Mercer, 1997, as cited by Allsopp et al., 2003);
- Building in multiple practice opportunities (Miller and Mercer, 1997, as cited by Allsopp et al., 2003) and time for review by students (Kameenui and Carnine, 1998, as cited by Fuson, 2003);
- Using reinforcement (e.g., earning verbal praise) (Mastropieri et al., 1991); and,
- Providing continual feedback (Miller and Mercer, 1997, as cited by Allsopp et al., 2003; Fuson, 2003; Blankenship, 1978, Schunk and Cox, 1986, as cited by Mastropieri et al., 1991).

Three of these elements of effective special needs instruction—modeling, mediated scaffolding, and feedback—are discussed in further detail below.

Modeling

Directly modeling both general problem-solving strategies and specific learning strategies using multisensory techniques has been shown to be useful with students having attention problems, cognitive-processing problems, memory problems, and metacognitive deficits, notes a summary of relevant research (Allsopp et al., 2003). A comparative study of 30 students suggests that direct modeling may be advantageous for students with slight mental retardation as well. One group of students who received direct modeling help (e.g., used blocks as physical manipulatives) and extra opportunities for purposeful practice employed "substantially fewer" inappropriate learning strategies than another group who didn't receive such support (Baroody, 1996, pp. 81-82). Furthermore, it was found that one or two direct-modeling demonstrations enabled such students to correct basic arithmetic procedural strategies and improve their proficiency (Baroody, 1996).

In addition, a review and synthesis of 30 studies on mathematics instruction for learning-disabled students found that modeling and demonstration with corrective feedback improved problem-solving accuracy and generalization skills by the students (Blankenship, 1978, as cited by Mastropieri et al., 1991). For example, an instructional model in which teachers solved a problem, verbalized how they did it, and left the problem as a reference model improved learning-disabled students' computational skills in seven different experiments (Smith and Lovitt, 1975, Rivera and Smith, 1987, 1988, as cited by Mastropieri et al., 1991).

Mediated Scaffolding

A review of 30 studies found several types of scaffolding strategies to be effective in improving learning-disabled students' mathematical achievement in grades K-6 (Mastropieri et al., 1991):

- Use of manipulatives teamed with pictorial representations (Peterson, Mercer, and O'Shea, 1988, as cited by Mastropieri et al., 1991);
- Multisensory approaches—mixing visual, auditory, and/or kinesthetic methods. Verbalization (a specific practice in which teachers and students repeat aloud problems, instructions, and solution steps) also improved students' mathematical performance, found a comparative study of 90 such students in grades 6 through 8 (Schunk and Cox, 1986, as cited by Mastropieri et al., 1991); and,
- Use of preorganizers (e.g., read problem, underline numbers, decide on the operation sign and problem type) and/or postorganizers (e.g., read problem, check operation, check math statement, check calculations, write labels) to support students when solving word problems (Mastropieri et al., 1991).

Feedback

Ongoing feedback is crucial with special-needs students. Such students require continual monitoring and feedback on their efforts to be successful, several studies and meta-analyses have found (Miller and Mercer, 1997, as cited by Allsopp et al., 2003; Fuson, 2003; Schunk and Cox, 1986, as cited by Mastropieri et al., 1991).

Addressing Specific Mathematics Disabilities

A synopsis of relevant research noted that four different kinds of mathematics disability have been identified (Geary, 1994, as cited by Fuson, 2003). They, and what the research suggested as useful strategies to address them, are as follows:

- Semantic memory disabilities: Students experience trouble with verbal and phonetic memory but may have normal visuospatial skills. Instruction that employs visual clues is most effective for these learners (Fuson, 2003).
- Procedural deficits: Students use less advanced methods overall. Conceptually based instruction is especially helpful for these students (Fuson, 2003);
- Visuospatial disabilities: Students struggle with concepts that use spatial relations, (e.g., place value). Instruction most helpful for these students includes extra cues to support visual processing, and focuses on methods that can be carried out in either direction (Fuson, 2003); and,
- Problem-solving deficits: Such students benefit from problem-drawing supports, including visual representations and manipulatives (Fuson, 2003).

English Language Learners (ELL)

In his review of the research on how race, ethnicity, social class and language might affect student achievement in mathematics, Secada (1992) found a relationship between the amount of proficiency in a given language and mathematics achievement (Fernandez and Nielson, 1986, Duran, 1988, Secada, 1991b, as cited by Secada, 1992). To support academic achievement for non-native speakers of English and other diverse learners, Secada recommended:

- Intervening early;
- Providing ongoing extra support materials and strategies;
- Using a student's native language for instruction;
- Using a structured curriculum or focus teaching on basic skills;
- Using small-group instruction, preferably in cooperative learning settings; and
- Carefully grouping students by specific ability, if necessary (Secada, 1992).

According to an article by McElroy (2005) teachers need to expand their teaching tools to assist ELL students in content areas such as math. The article describes a website Colorin Colorado that teachers can use to work with ELL students. Material specific to ELL are presented along with teaching tips. While focused more on the language learning of ELL, several recommendations are made by Goldenberg (2006). The instructional practices seen as having a positive impact specific to math include:

- clear instructions and expectations
- additional opportunities for practice
- extended explanations

In a well presented case, Abedi (2004) reports on the difficulty of assessment of math and reading for ELL, especially as related to the No Child Left Behind (NCLB) act. Factors are presented as issues including the sparse ELL populations in some states, subgroup lack of stability, linguistic complexity of assessment tools and lower ELL baselines requiring greater gains. The implications are that unless these issues are considered, schools with large ELL populations face unfair and undue pressure under NCLB. In a similar vein, the American Federation of Teachers (AFT, 2006) point out that NCLB challenges faced by ELL students in math and reading include defining ELL subgroups and the paucity of natural language assessment. The AFT identified four changes needed:

- appropriate tests for ELL students
- relevant and valid testing of ELL students in English
- clarifying assessment of proficiency versus math and reading skills
- clarifying existing policies regarding ELL immigrant and non-immigrant groups

Differentiated Instruction

Ample research has concluded that students find more success and satisfaction in school if they are taught in ways that are responsive to their readiness levels (e.g., Vygotsky, 1986), interests (e.g., Csikszentmihalyi, 1997), and learning profiles (e.g., Sternberg, Torff, and Grigorenko, 1998) (as cited by Tomlinson, 2000). Differentiated instruction is how this translates into classroom practice.

> Every elementary classroom holds a wide range of learners: In most elementary classrooms, some students struggle with learning, others perform well beyond grade-level expectations, and the rest fit somewhere in between. Within each of these categories of students, individuals also learn in a variety of ways and have different interests. To meet the needs of a diverse student population, many teachers differentiate instruction (Tomlinson, 2000).

Differentiated instruction involves varying ones' teaching according to each learner in either (1) content, (2) instructional process, (3) students' products (e.g., papers, projects, computer models), and/or (4) learning environment (e.g., cooperative learning in small groups, grouped by ability) (Tomlinson, 2000). By definition, differentiated instruction always involves ongoing assessment linked to instructional decisions and planning (Tomlinson, 2000). Because differentiated instruction focuses on each learner's varying needs, it is especially well suited for special-needs students.

The quality of the curriculum and instruction used during differentiation is crucial. High quality curriculum focuses on what experts deem the most essential mathematical concepts and skills. High quality instruction incorporates lessons, tasks, and materials designed to ensure that students (1) grapple with essential concepts and skills, (2) find the learning experiences relevant and interesting, and (3) are engaged in active learning experiences (Tomlinson, 2000).

Research has also shown that flexible groupings can improve the mathematical achievement of special-needs students (Slavin, Madden, and Leavey, 1984, as cited by Mastropieri et al., 1991; Mastropieri et al., 1991; Secada, 1992; Slavin, Madden, Karweit, Livermon, and Dolan, 1990, as cited by Secada, 1992). Teachers can use flexible grouping to deliver a variety of differentiated learning environments in their classrooms, including small workgroups, cooperative learning groups, cross-grade groups, between-grade groups, grouping by ability for guided or independent practice, as well as whole class, and individual practice settings (Tomlinson, 2000).

Furthermore, Burris, Heubert and Levin (2006) found that completers of advanced math courses increase in all heterogeneous grouped students including minority and low SES students. The same conclusion was reached for all students at whatever initial achievement level. Initial high achievers performed the same as counterparts in homogeneous groups. Rates of participation and test scores improved in all groups.

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In yet another piece (Zehr, 2006), the changes brought by NCLB are debated as to their positive versus negative effects. The major point made is that many schools may be responding to NCLB in math and reading by narrowing the curriculum to focus only on test scores. With respect to the No Child Left Behind legislation, Kim and Sunderman (2005) suggets that use of the mean proficiency score may have a disparate effect on schools with low income children. Accountability rules for the sub groups can over identify racially diverse schools as failing to meet proficiency levels. The use of multiple indicators of performance may produce a more accurate assessment of student proficiency. Multiple measures could include a measure of improvement.

Much debate has surrounded the achievement issues in public versus private education. Charter schools, educational vouchers, etc., have been the responses of individuals and groups concerned about public school performance issues. Utilizing mathematics data from the 2003 NAEP data, Lubienski and Lubienski (2006) analyzed differences among students from public, charter and other types of private schools. They found that once demographic information was statistically controlled (ethnicity, gender and SES), no significant differences favoring private over publivc entities were found. In fact, public school fourth graders scored significantly higher that than their private school counterparts. Their finding validate the importance of other in-school factors such as curriculum, teacher competence, etc.

How McGraw-Hill's Math Connects Series Reflects the Research on Ensuring Mathematical Proficiency for All Learners

McGraw-Hill's *Math Connects* series is designed to promote math proficiency for all learners. While high quality teaching practice promotes every student's learning, quality of instruction assumes essential importance for the learner with special needs.

The Teacher's Editions of McGraw-Hill's *Math Connects* series, using an approach of differentiated instruction and extra supports, provides teachers with the tools they need for addressing the distinct needs of the diverse learners one finds in a K-Algebra classroom. This approach is extended in the Student Edition and ancillary materials, which support many different learning styles and ability levels to suit a range of learners. Thus, the program can meet the special needs of students who are low performing for reasons of specific disabilities (e.g., mental, physical, behavioral, learning, and/or lack of language skills), as well as gifted and advanced learners.

Specific research-based characteristics of McGraw-Hill's *Math Connects* series that especially benefit special-needs students include the following:

- Sets clear goals for students in the Student Editions
- Teaches the "big idea" conceptual base before moving to specific details of procedures
- Begins with the concrete, then progresses through the pictorial to the abstract.
- Models concepts and procedures
- Provides mediated scaffolding, including the following:
  - Uses manipulative materials and pictorial representations wherever appropriate
  - Uses multi-sensory instructional approaches
  - Teaches procedures in small steps, with correctly worked problems made available to students or modeled
  - Guides student use of pre- and post-organizers
- Builds on previous knowledge and uses appropriate context for more meaningful learning
- Uses verbal and written language to describe ideas and procedures, because symbolic representation by itself is usually insufficient
- Provides meaningful practice, student review, and teacher feedback
- Guides teachers to differentiate instruction by offering
  - Multiple teaching strategies and modalities
  - Extra support for struggling students
  - Multiple levels of practice and problem solving challenges
- Provides targeted resources for English language learners, including multilevel language support and grouping suggestions to teachers.

As the research states, it is important that all students understand instructional goals. To this end, McGraw-Hill's *Math Connects* series provide unit, chapter, and lesson goals throughout the program. In the early grades, chapter objectives are noted in the Chapter Planner page in the Teacher Edition.

Once clear learning objectives are set forth, teachers are guided to reach all learners at their individual levels throughout the program, as the research purports. Differentiated instruction is built into the Teacher's Edition, offering multiple strategies and modalities to reach all learners. Alternate activities and teaching strategies are integrated to give teachers the tools to make content accessible for students at all levels. The Reaching All Learners section of the Chapter Planner prompts in the Teacher's Edition supply further resources for students below, on or at level and those needing language support (ELL). Scaffolded instruction is incorporated in grades 3-6, breaking down problems into small steps so that students at all levels can become independent problem solvers.

This instruction approach, aimed at meeting the needs of all learners, is continued in the Student Edition. Instruction and practice in the Student Edition ensures that mathematical content is not too difficult for students, as asserted in the research. McGraw-Hill's *Math Connects* series is in line with the research that math instruction should build upon previous knowledge and use appropriate context for more meaningful learning. Lessons build on the objectives presented through out the program and connect to real world math. In the Skills Trace section of the Teacher's Editon, vertical alignment of skills is clearly shown.

McGraw-Hill's *Math Connects* series provides all the resources necessary to prepare English language learners for success in math. Language support is integrated throughout the program with multilevel strategies to engage non-English speakers at three levels of acquisition – Preproduction,Early Production and Speech Emergence, and Intermediate and Advanced Fluency. Teachers receive grouping suggestions at the beginning of every chapter and with diagnostic tests to ensure language learners are placed appropriately and get ample opportunity for small group work, as suggested by the research. Intervention prompts provide strategies for teachers to ensure these students are attaining adequate mathematical proficiency, Daily lessons in the Teacher's Edition include Check Points that ensure early intervention is made available for English language learners needing extra support.

Again, lesson 12.5 in the Grade 1, Chapter 12 text illustrates some key research points. In the Teacher's Edition, a Differentiated Instruction section provides small group options for students below grade level and ELL students. Independent work options are given for students on level and above level including the use of technology. As well, three different problem solving approaches are presented (number sentences, tables and making a model). In Algebra (Chapter 3, lesson 3-1), options for understanding ordered pairs are presented. For struggling and special needs students, a connection to music option is given (pairing notes to their number of beats for those musically inclined). A Scaffolding Questions section is presented, as well as exercises to illustrate relations. For the baseball example noted previously (years, homeruns and strikeouts), students are asked to think about the relationship of homeruns to strikeouts over the years.

McGraw-Hill's *Math Connects* series evidences an array of instructional characteristics that research suggests are beneficial to special-needs students. Teachers have the resources and guidance in the Teacher's Edition to effectively reach all learners. The Student Edition gives students many opportunities for practice at their individual levels. Other key ancillary materials ensure that all students are geared for success in the math classroom. The program follows a strategy of differentiated instruction and practice to address the needs of the diverse learners in grades K-Algebra.

Assessing Mathematical Proficiency to Inform Instruction

Research suggests that assessments greatly impact what and how teachers teach, and what their students learn. A large-scale study of grades 4-12 found that tests—both standardized and those in textbooks—

have an “extensive and pervasive influence on math instruction” (Madaus, West, Harmon, Lomax, and Viator, 1992, p. 1, as cited by Mathematical Sciences Education Board [MSEB], National Research Council, 1993). As well, Ryan, Ryan, Arbuthnot and Samuels (2007) suggest that motivational patterns of students may lead to less effective test-taking strategies.

Given the key role that assessment plays in instruction, the National Council of Teachers of Mathematics' Principles and Standards for School Mathematics (2000) has recommended that “assessment should support the learning of mathematics and furnish useful information to both teachers and students” (NCTM, 2000, p. 22). It continues:

> Assessment should be more than merely a test at the end of instruction to see how students perform under special conditions; rather, it should be an integral part of instruction that informs and guides teachers as they make instructional decisions. Assessment should not merely be done *to* students; rather, it should also be done *for* students, to guide and enhance their learning (NCTM, 2000, p. 22).

The Mathematics Sciences Education Board (MSEB) of the National Research Council's (NRC) Center for Education summarized the key role of assessment even more succinctly: “Assessment is the guidance system of education just as standards are the guidance system of reform” (MSEB, NRC, 1993, p. 2).

Principles and Standards for Assessing Students' Mathematical Proficiency

Based on its synthesis of relevant research, the NCTM published Assessment Standards for School Mathematics (1995), which identified six standards for “exemplary mathematical assessment” that supported its standards for teaching and learning mathematics in kindergarten through grade 12 (NCTM, 1995, as cited by Wilson and Kenney, 2003, p. 53). Three of these standards are elaborations of core principles of the MSEB, based on its own review and synthesis of research on national efforts to reform the teaching and learning of preK-12 mathematics (MSEB, NRC, 1993). As summarized in Principles and Standards for School Mathematics (NCTM, 2000), assessment should:

1. “Reflect the mathematics that all students need to know and be able to do” (NCTM, 2000, p. 22). This is similar to the MSEB's “Content Principle,” which stressed assessment that reflects “the mathematics that is most important for students to learn” (MSEB, NRC, 1993, p. 1).
2. “Enhance mathematics learning” (NCTM, 2000, p. 22). This is similar to the MSEB's “Learning Principle,” which also noted that assessment should “support good instructional practice” (MSEB, NRC, 1993, p. 1).
3. “Promote equity” (NCTM, 2000, p. 22). This is similar to the MSEB's “Equity Principle,” which explained, “Assessment should include opportunities to allow every student to demonstrate the extent of their mathematical knowledge.” (MSEB, NRC, 1993, p. 1). By “every student,” the MSEB meant each child, regardless of prior knowledge or societal background.
4. “Be an open process” (NCTM, 2000, p. 22).
5. “Promote valid inferences about mathematics learning” (NCTM, 2000, p. 22).
6. “Be a coherent process” (NCTM, 2000, p. 22).

Characteristics of Effective Assessment

Research has identified some key characteristics of effective approaches to assessing students' mathematical proficiency and understanding. In general, such assessments need to record "genuine accomplishments in reasoning and in formulating and solving mathematical problems," not just the ability to memorize or recognize correct answers, noted a retrospective analysis of the items in the 1990 National Assessment of Educational Progress (NAEP) test (Silver, Kenney, and Salmon-Cox, 1991, as cited by MSEB, NRC, 1993, p. 31-32).

No single form of assessment can adequately measure the breadth and depth of students' knowledge. In general, the richer in quantity and variety of evidence used for assessment, the more valid a teacher's inferences of a student's understanding and proficiency are likely to be, concluded the NCTM's Assessment Standards Working Groups and other researchers (NCTM, 1995; Ames, 1992, as cited by Wilson and Kenney, 2003; Wilson and Kenney, 2003). Multiple types of assessments enable teachers to look for "convergence" and "common conclusions" (Linn, Baker, and Dunbar, 1991, Baxter, Shavelson, Herman, Brown, and Valadez, 1993, as cited by MSEB, NRC, 1993, p. 133; Wilson and Kenney, 2003). Furthermore, several studies and syntheses of research on assessment in grades K-12 have found that in order to effectively measure mathematical knowledge, tasks used for formative assessment should not only be varied but should also be nonroutine (i.e., different from what students have previously experienced), offer reasonable challenge, and focus on meaningful aspects of learning (Ames, 1992, McLeod, 1992, MSEB, NRC, 1993, as cited by Wilson and Kenney, 2003).

Useful instruments for formative assessment include oral comments; written papers, including drafts and polished work; journal entries; drawings; computer-generated models; out-of-class long-term projects; and different types of small-group work, researchers have concluded (NCTM, 1995; NCTM, 2000; Wilson and Kenney, 2003).

Teachers generally believe that what they see and hear as their students engage in mathematical learning and practice, along with other documentation of students' performances, are the most useful components of formative assessment. For example, a study of 269 teachers of grades 1-4 mathematics rated their own observations and students' performances as the two most important elements of assessment 3. Similarly they rated teacher observations and students' performances as their most often-used assessment practices (Adams and Hsu, 1998). However, the same study also showed that grades 1-4 mathematics teachers believe that using a variety of assessment methods is valuable as well, and reported using more than half a dozen methods either "very often" or "almost always" in their formative assessments. These additional methods included watching how students applied mathematics, how they explored problem solving, use of direct questioning, and class discourse/discussion (Adams and Hsu, 1998, p. 177).

Larger pieces of work—such as performance tasks, projects, and portfolios—allow students to demonstrate how they have grown in their mathematical power (NCTM, 1995). Portfolios are considered valuable for summative assessments precisely because they measure different constructs than standardized tests (Reckase, 1997, as cited by Wilson and Kenney, 2003). Having students participate in choosing which work samples to include in their portfolios prompts them to reflect on and monitor their own learning, and stimulates teachers to reflect on their own practice as well, several studies and research reviews have noted (Columba and Dolgos, 1995, Daro, 1996, Mills, 1996, Yancey, 1996, as cited by Wilson and Kenney, 2003).

In textbook-based learning, adjunct questions—when used to focus students' attention on the concepts presented in the text—were found to enhance learning (Crooks, 1988, as cited by Black and Wiliam, 1998).

Additionally, a recent synthesis of research on assessing mathematical proficiency found that classroom assessment should include adequate dialogue between teachers and students. Moreover, teachers need to ask higher-level questions, which "has not been the norm," the synthesis emphasized (Wilson and Kenney, 2003, p. 55).

"Assessment is to be seen as a moment of learning, and students have to be active in their own assessment and to picture their own learning in light of what it means to get better," noted Zessoules and Gardner in their review of research (Zessoules and Gardner, 1991, as cited by Black and Wiliam, 1998, p. 30). Engaging students in measuring their own and their peers' mathematical knowledge is effective, research has found. For example, two quantitative studies involving more than 700 students aged 8-14 years indicated that student self assessment and peer assessment of their mathematical understanding did help enhance their learning—as long as they understood the learning goals and assessment criteria, and had an opportunity to reflect on their work (Fontana and Fernandes, 1994, Frederiksen and White, 1997, as cited by Black and Wiliam, 1998). The NCTM's Assessment Standards Working Groups concurred with this finding (NCTM, 1995). Similarly, higher academic achievement resulted "from combinations of self-assessment and peer assessment" by students, found a study of the methods' effectiveness when used by young children (Higgins, Harris, and Kuehn, 1994, as cited by Wilson and Kenney, 2003, p. 57).
Zessoules and Gardner also concluded that assessment that helps students "develop reflective habits of mind" contributes to their learning (Zessoules and Gardner, 1991, as cited by Black and Wiliam, 1998, p. 29).

Training students in aspects of self assessment may give them a learning advantage. For example, one study found that fifth and sixth graders, after being trained to monitor their own performance on mathematical problems, were more successful than those who had no such training. They could solve more complex problems and also succeed more quickly. They seemed to do better "not because they could use particular strategies more effectively, but because they started by considering the possibilities of using different strategies before proceeding" (Delclos and Harrington, 1991, as cited by Black & Wiliam, 1998, p. 27). Reviews of the literature on assessment have also concluded that an assessment must reflect the instructional setting(s) in which the content was taught. For example, if students' work was done in group settings, the assessment should reflect the value of group interaction (Davidson, 1985, Good, Mulryan, and McCaslin, 1992, as cited by MSEB, NRC, 1993).

A meta-analysis of 40 studies on the effects of in-class testing showed that frequent testing—but no more than one or two per week—improved student achievement. Several short tests proved more effective in improving student achievement than fewer long tests (Bangert-Drowns, Kulik, and Kulik, 1991, as cited by Black and Wiliam, 1998; Wilson and Kenney, 2003). In addition, student self assessment has been demonstrated as effective when they are made a part of everyday class instruction. Significant student achievement was attained by having "students self-assess on a regular (mainly daily) basis," found a study of 356 students aged 8-14 years (Fernandes and Fontana, 1996, as cited by Wilson and Kenney, 2003, p. 57).

To achieve coherence, a recent synthesis of research on assessing mathematical learning concluded, assessment must be aligned with both the curriculum and the instruction, it must match the instructional purpose, and the phases of assessment must fit together in a meaningful progression (Wilson and Kenney, 2003).

Formative Assessment

Many studies confirm that formative assessments—measures of students' learning of mathematical concepts and procedures that teachers use to guide ongoing instruction in the classroom—significantly enhance student achievement. For example, a recent meta-analysis of 250 studies found that "the research shows conclusively that formative assessment does improve learning," (Black and Wiliam, 1998, p. 61.) Furthermore, "the gains in achievement appear to be quite considerable, amongst the largest ever reported for educational interventions" (Black and Wiliam, 1998, p. 61). A mean effect size of 0.7, as reported by Fuchs and Fuchs' (1986) analysis of 21 studies on assessment of preK-12 children, is cited as representative (Black and Wiliam, 1998).

To illustrate such a large gain attributed to formative assessment, if accomplished nationwide, it would be the equivalent of raising students' mathematics-attainment score on the Third International Mathematics and Science Study (TIMSS) from "average" (where the U.S. was ranked) up into the "top five" (after Singapore, Korea, Japan and Hong Kong) (Beaton et al., 1996, as cited by Black and Wiliam, 1998, p. 61).

Research also shows that students have higher achievement if their learning goals are "dynamic," that is, if the goals are adjusted or amended with corresponding changes in instruction in light of measured progress (Fuchs, 1993, as cited by Black and Wiliam, 1998, p. 44).

Formative assessment by teachers offers benefits beyond raising test scores, noted the NCTM's Assessment Standards Working Groups: "Continuous assessment of students' work not only facilitates their learning of mathematics but also enhances their confidence in what they understand and can communicate" (NCTM, 1995, p. 14).

Teachers need interpretive frameworks to help them coordinate the many separate bits of assessment information into a usable result, revealed a study of elementary teachers' practices. The researchers found that currently, such frameworks are generally absent (Bachor and Anderson, 1994, as cited by Black and Wiliam, 1998). Results from Fuchs and Fuchs's meta-analysis of 21 studies on assessment of preK-12 students suggest that interpretive frameworks would improve students' mathematical achievement. It was found that teachers who produced graphs of individual children's progress and used them to guide further action reported greater student gains than teachers who did not make or use progress graphs, with mean effect sizes of 0.70 and 0.26, respectively (Fuchs and Fuchs, 1986, as cited by Black and Wiliam, 1998).

According to a meta-analysis of 58 experiments of feedback's effect on instruction, it is the quality of feedback from assessments that has the most influence on students' performance gains (Bangert-Drowns, Kulik, and Kulik, 1991, as cited by Black and Wiliam, 1998). Feedback from teachers proved to be most effective when it provided specific help on correcting students' errors and improving their strategies, yet kept the overall focus on deep conceptual understanding, found a related meta-analysis (Bangert-Drowns, Kulik, Kulik, and Morgan, 1991, as cited by Wilson and Kenney, 2003).

Furthermore, feedback appears to be most effective when it focuses on the task and not on the student. Studies have found that students who received feedback that was focused on learning goals (task-oriented) had higher achievement, sought help more often when they needed it, and generally had a more positive attitude about themselves and towards learning than peers who received feedback that was focused on performance goals (ego-oriented) (Butler, 1988, Butler and Neuman, 1995, as cited by Black and Wiliam, 1998). A meta-analysis of relevant studies reached similar conclusions (Bangert-Drowns, Kulik, Kulik, and Morgan, 1991, as cited by Wilson and Kenney, 2003). Studies and syntheses of relevant research have also concluded that teachers should routinely perform instructional follow-up to assessments in the form of classroom discourse that "revisits, extends, and generalizes" interesting and/or key types of mathematical problems (MSEB, NRC, 1993, pp. 11, 78).

Feedback to students after assessment must be timely. As part of their meta-analysis, Bangert-Drowns, Kulik, and Kulik found that immediate feedback was more effective than feedback that was delayed by even a day after the assessment (Bangert-Drowns, Kulik, and Kulik, 1991, as cited by MSEB, NRC, 1993).

In the process of gauging students' knowledge, educators must be careful that their methods are equitable and don't contain hidden biases or assumptions concerning students' capabilities. For an assessment to be equitable, it must provide each and every student with the opportunities to demonstrate his or her level and/or range of mathematical knowledge, found a recent synthesis of research on assessing mathematical learning (Wilson and Kenney, 2003). This may mean, for example, that students receive partial credit for successfully completing parts of a mathematical task (Wilson and Kenney, 2003).

In addition, to ensure equity, assessment tasks must utilize "a context that is familiar to all students," regardless of their prior knowledge or social background, found another review of relevant research on mathematical learning (MSEB, NRC, 1993, p. 50). One way to achieve this, developed by the NRC's Learning Research and Development Center, is to first introduce the context in a separate lesson prior to the assessment (Learning Research and Development Center, University of Pittsburgh, and National Center on Education and the Economy, 1993, as cited by MSEB, NRC, 1993).

How McGraw-Hill's Math Connects Series Reflects the Research on Assessing MathematicalProficiency

Consistent with the research, McGraw-Hill's *Math Connects* series integrates a comprehensive, coherent assessment plan, featuring multiple, frequent opportunities and diverse methods to assess students' mathematical learning. The program expressly addresses diagnostic assessment (for placement or readiness), formative assessment (to inform ongoing instruction), and summative assessment (evaluations of performance for a grade). These different types of assessments are clearly identified in the materials and guidance found in the Teacher's Editions of McGraw-Hill's *Math Connects* series.

In each unit in the Teacher's Edition is a section titled "Assessment Options" . This lists all of the assessments for the unit, categorizing them as diagnostic, formative assessment, or summative assessment. It also suggests resources suitable for each assessment, correlated to that unit's particular content. Thus, teachers receive varied assessment resources for instructional units in advance, as well as chapter by chapter within units.

In McGraw-Hill's *Math Connects* series, assessment begins with a pre chapter diagnostic readiness section of information on options for diagnostic assessment. Macmillan/McGraw-Hill Math provides teachers with strategies for reaching all learners – including those in need of language support – as they observe students, engage them in assessment dialogues, and have them read and write about math. Using these assessment techniques, teachers can ensure an equal opportunity for all students to demonstrate their mathematical proficiency.

McGraw-Hill's *Math Connects* series' comprehensive, coherent assessment plan reflects the research on assessment by deliberately emphasizing the use of formative assessment to inform and improve ongoing classroom instruction and by providing both the assessment instruments and guidance. Frequent, multiple assessment opportunities for students and related resources for teachers are present in every lesson of every unit at every grade level. The program's Teacher's Edition, Assessment Guide, Intervention Handbook, and ESL Activity Guide promote frequent use of these assessment options by teachers, providing the extra resources and guidance to do so with confidence. In short, McGraw-Hill's *Math Connects* series provides a wealth of material for teachers and students to ensure valid, dynamic and comprehensive assessment to support ongoing instruction for all students.

Professional Development to Improve Mathematics Proficiency

Teachers are central to students' success in mathematics, especially in the elementary grades. Teachers serve in a variety of critical roles—including classroom manager, content presenter and demonstrator, activity coordinator, project director, and learning guide—that help define students' educational experiences with mathematics. "Students' understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school" (NCTM, 2000, pp. 16-17). Successful mathematics learning by students, therefore, depends on the skills, knowledge, and practices of their teachers.

Professional development is the primary vehicle for improving teachers' skills, knowledge, and practices in mathematics and instruction. For teachers to "understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks," notes the NCTM in Principles and Standards for School Mathematics, they "must have frequent and ample opportunities and resources to enhance and refresh their knowledge" (NCTM, 2000, p. 17). In support, Hill, Rowan and Ball (2005) found that teacher's math content knowledge related to student achievement grades 1-3

This view of the importance of professional development is supported by research on its cost-effectiveness. In their review of the research, Greenwald, Hedges, and Laine (1996) concluded, "Increased spending on teacher education has a greater positive effect on student learning than increased spending on other school variables" (as cited by Stigler and Hiebert, 1999, p. 146).

With the NCTM's vision of effective mathematics teaching and learning in mind, Mewborn (2003) reviewed more than 20 years of research on professional development in mathematics instruction. Based on her synthesis of that research, she recommended that such professional development give teachers opportunities to learn mathematics in the ways they are now expected to teach it to students, that is, for conceptual understanding as well as procedural proficiency. She further concluded that professional development should help teachers gain insight into the conceptual underpinnings of topics and the interconnections among topics of the mathematics they teach. The body of evidence “overwhelmingly suggests,” summarized Mewborn (2003, p. 49), that enhancing teachers’ mathematical knowledge is “a crucial part of learning to teach differently” (Campbell and White, 1997; Heaton, 1994, 2000; Kilpatrick, Hancock, Mewborn, and Stallings, 1996, as cited by Mewborn, 2003).

For instance, a two-year study of five middle-school teachers teaching rational numbers, quantity, and proportional reasoning found teachers’ practices changed as their content knowledge increased and deepened (Sowder, Philipp, Armstrong, and Schappelle, 1998, as cited by Mewborn, 2003). Also, a comparative study of 21 upper elementary-grade classrooms in California (following its adoption in 1992 of a new problem-solving Mathematics Framework), found that professional development designed to enhance teachers’ understanding of mathematics contributed to greater opportunities for students to engage with numeric representations in ways that helped students build conceptual understanding (Gearhart, Saxe, Seltzer, Schlackman, Ching, Nasir, Fall, Bennett, Rhine, and Sloan, 1999).

Schifter (1998) found that effective professional development has teachers “struggle with important mathematical ideas, justify their thinking to peers, investigate alternative solutions proposed by others, and reconsider their conceptions of what it means to do mathematics” (as cited by Mewborn, 2003, p. 49). Schifter’s research explored the ability of teachers to readily translate what they learned during inservice into classroom practice. Professional development in which teachers actively learned new ideas (as would be expected of their students) better enabled those teachers to turn the training into improved instructional practice, compared to teachers who were simply told the new ideas, Schifter reported.

Research also suggests that professional development should help teachers encourage their students to explore multiple ways to accomplish mathematical tasks. For example, a review of research found that “richer, more conceptual” learning opportunities become available when teachers encourage students to explore the relative advantages of different mathematical approaches to solving a problem (Hiebert, Fennema, Carpenter, Fuson, and Wearne, 1997, as cited by Stigler and Hiebert, 1999). And findings by Ball and Bass (2000) suggest that teachers must be able to “decompose” a math task, such as working backwards to show students its many possible ways of enactment. However, such decomposition works only if it is sufficiently granular—“if its logical steps are small enough to make sense for a particular learner or class,” based on what they already know (Ball and Bass, 2000, pp. 98-99). Professional development can provide that granularity of understanding to teachers.

To implement the kind of math instruction recommended by the National Council of Teachers of Mathematics effectively, a review of research concluded, changes in both beliefs and practice are necessary—and they are iterative (Goldsmith and Schifter, 1997, as cited by Nelson, 1997). It is not clear yet, however, which is more efficient: changing beliefs first or changing practices first. It seems reasonable that changes in belief may need to precede changes in practice (Schifter and Fosnot, 1993, as cited by Nelson, 1997). However, another review of the research reveals that in some cases, changes in practice have “jump started” the process for teachers (Franke, Fennema, and Carpenter, 1997, as cited by Nelson, 1997, p. 11).

Professional development focused on students’ mathematical thinking can have a positive impact on student achievement, found a comparative study in California of 21 upper elementary-grade classrooms covering the topic of fractions (Gearhart et al., 1999). In the study, one group of teachers received inservice during which they added to their own knowledge base of mathematics and assessment, and also engaged in peer dialogue and support. A second group of elementary teachers only had a collegial support group for dialogue and reflection, while a third of teachers group had no inservice at all. Students whose teachers were in the first group (receiving inservice and support) had more opportunities to conduct conceptual discussions and build their understanding of fractions and were more likely to experiece gains in performance on problem-solving tasks than students of the other teachers. Specifically, the study found that for each 1-unit increase on a 4-point Conceptual/Assessment Opportunity scale, there was an increase of .087 units on a 13-point Problem-Solving scale (Gearhart et al., 1999, following procedures developed by Stein and Lane [1996] and Stein, Grover, and Henningsen [1996] for coding mathematical tasks and relating them to student outcomes). Thus, concluded the researchers, professional development for teachers that is expressly designed to give students more opportunities to engage and explore mathematical concepts and problem solving results in learning gains (Gearhart et al., 1999).

Another study of California's mathematics reform initiative noted that gains in student achievement were maximized when the teacher professional development, curriculum, and student assessment measures were all aligned with the prevailing policies of reform (Cohen and Hill, 1998, as cited by Mewborn, 2003).

Among the best-supported conclusions in the literature on educational improvement is this: "Almost all successful attempts to improve teaching have involved teachers working together to improve students' learning" (Stigler and Hiebert, 1999, p. 135). A comprehensive review of the research on professional development summarizes the relevant studies and their findings (Darling-Hammond, 1998, as cited by Stigler and Hiebert, 1999). For example, a comparative study of teachers at an urban elementary school found that professional development was more effective in helping teachers change their practice when it included collegial support (Kazemi and Franke, 2000, as cited by Mewborn, 2003).

However, receiving collegial support alone is not enough, found a comparative study of the effects of different types of inservice on 21 elementary-grade classrooms. Professional development programs that only supplied collegial support were not as effective as those that also concomitantly focused on curriculum, mathematics, and/or children's mathematical thinking (Gearhart et al., 1999).

Davis and Krajcik (2005) provide nine design heuristics related to professional development from a NSF study to promote student learning (science related but non grade specific):

1. engage students in the topic specific phenomena (physical experiences, etc.)
2. utilize scientific instructional representations (analogies, models, diagrams, etc.)
3. anticipate understanding and dealing with student ideas about science (age specific) – incorporate into teaching
4. engage students in questioning – provides focus for teaching
5. engage students to collect and analyze data and observe
6. engage students to design their own inquiry
7. explain using evidence
8. promote scientific communication
9. develop subject matter expertise of teachers

The conclusion that teaching is clearly a system of unique features that have to work together to support learning goals was stated by Hiebert, Stigler, Jacobs, Givvin, Garnier, Smith, Hollingsworth, Manaster, Wearne, and Gallimore (2005). They add that all of the features must be implemented effectively with students and that no one single feature added could be said to be individually effective.

International Perspective

Japanese students have academically outperformed U.S. students in science and mathematics, according to Third International Mathematics and Science Study (TIMSS) results in 1995, 2000, and 2003. Stigler and Hiebert's (1999) analysis of TIMSS videotapes of classrooms in Japan, Germany, and the U.S., as well as Fernández' (2002) exploration of Japan's "lesson study" form of professional development, reveal likely explanations for this learning advantage.

First and foremost, U.S. elementary mathematics teaching was very limited and focused mainly on a narrow band of procedural skills, showed the TIMMS' videotapes of classrooms. Comparatively, Japanese teachers taught math "in a deeper way," for conceptual understanding. Thus, U.S. students spent most of their time "acquiring isolated skills through repeated practice" while Japanese students spent as much time "solving challenging problems and discussing math concepts as they [did] practicing procedures" (Stigler and Hiebert, 1999, p. 11). Furthermore, the U.S. students typically practiced previously demonstrated procedures on many simple problems. Conversely, Japanese students typically developed a variety of advanced techniques by themselves to use for solving a few challenging problems (Stigler and Hiebert, 1999).

"Lesson study" is a method of collaborative professional development in Japan that focuses on a specific goal for students, which is addressed through instruction. Teachers meet regularly to discuss and design detailed lessons to meet an agreed-upon goal, then also come together to watch and take notes as the lessons unfold in real classrooms. When they next meet, the educators comment, critique and collaborate to improve each lesson. The process is iterative, and a lesson might be continually honed for months or years. Lesson study allows teachers to deepen their understanding of both mathematics and of creating lessons that help students to come, by multiple ways, to a similar deep understanding, explains an empirical study of the feasibility of a similar approach to professional development in the U.S. (Fernández, 2002). Fernández' empirical study, conducted over the course of three years with 33 K-8 teachers engaged in mathematics lesson study, concluded that this approach to professional development did promote teachers' exploration of their own understanding of both mathematics and pedagogy. According to Stigler and Hiebert (1999), the lesson study approach to professional development holds promise for improving teaching and learning in the U.S. These researchers point out that lesson study directly improves instruction in context by focusing "on the lesson as the unit to be analyzed and improved" (Stigler and Hiebert, 1999, pp. 122). Moreover, since improvements are curriculum-based, professional development based on Japan's lesson-study model "uniquely allows" teachers to devote their time to those improvements that best align with local and state standards and for which they are held accountable (Stigler and Hiebert, 1999, pp. 154).

The collaborative nature of lesson study helps counter-balance individual teachers' self-critiquing with "the idea that improved teaching is a joint process and not the responsibility of any individual" (Stigler and Hiebert, 1999, p. 124-125). And finally, the TIMMS study emphasizes that lesson study is a long-term professional development model of gradual, but continuous, improvement (Stigler and Hiebert, 1999).

In contrast, Wang and Lin (2005) state that there is not enough evidence in the existing literature (international comparative) to say conclusively that a strong positive relationship exists between professional development, implementing a curriculum or pedagogical features (standards, organization, teacher content knowledge, etc.) and math performance. The relationships are complex and interactive. All features working together may be necessary to directly influence performance.

The above perspective was confirmed by Desimone, Smith, Baker and Ueno (2005). They identified perceived barriers to use of a conceptual approach in US math education that have been used to explain why U. S. students do not do as well as international students on math tests:

1. too much individualism and autonomy of teachers (both style and implementation of known strategies to lead to expected reforms
2. more use of conceptual strategies will diminish use of computation – seen as a dichotomy as opposed to complementary
3. U. S. teachers - use conceptual only with high achievers, computation for low achievers
4. too large class sizes in US schools – keep conceptual approach at low use
5. only teachers with a lot of math content and math pedagogy have the ability to use conceptual approach

Their results failed to confirm any of the above as evidence of differences in U. S. versus international student performance.

How McGraw-Hill's Math Connects Series Reflects the Research on Professional Development in Mathematics Teaching

McGraw-Hill's *Math Connects* series provides multiple avenues of assistance for the professional development of teachers. While it focuses primarily on teaching K-Algebra students and does not purport to be a comprehensive professional development program, the series does include several valuable professional development features in the Teacher's Edition and ancillary materials. These features help teachers develop greater content knowledge and deeper understanding of key math concepts as they teach their students, as recommended by research.

Consistent with the literature regarding the need for pedagogical practice, each chapter in the Teacher Editon series for K-6 provides a Skills Trace section (regarding the specific skills considered), showing what students should have learned for the previous grade, what they will learn in the chapter being studied and what will be considered in the nexy grade. As well, there is a Chapter Planner showing objectives, vocabulary, lesson resources, technology and ways to reach all learners. In some chapters, a section What the Research Says is included. For example, in Grade 2, Chapter 5, an article is abstracted regarding differentiating instruction in mixed ability classrooms. In the Algebra text, a research article is abstracted considering the interaction of algebraic representation and natural language habits. Responding to the literature regarding increasing content skills of teachers, each chapter in the Grade 7 and Algebra texts provide assistance in understanding the content. For example, in the Algebra text, hints on understanding linear equations are provided.

Research indicates the benefits of exposing teachers to different approaches to solve problems. In every problem-solving lesson an alternative strategy is given. This provides the teacher with instruction for solving the problem using another approach and enables the teacher to open the door for discussion of how there are always alternative strategies to solve a problem.

To further improve professional practices, McGraw-Hill's *Math Connects* series offers Mathematics, Yes!, which combines video and interactive Web content to model instructional strategies and provide a framework for self-assessment by teachers and for assessment by mentors or peer coaches. There is also an opportunity for teachers to share ideas and experiences with each other, benefiting from the collegial support that research indicates improves educators' teaching.

Because McGraw-Hill's *Math Connects* series provides a carefully-structured sequence of lessons and ample support materials that model and guide teachers to exemplary instructional practice, the program can serve as a component of a district's own comprehensive program of professional development in mathematics instruction. Such a staff development program might include presentations and demonstrations by staff developers, readings for development of conceptual understanding and deeper content knowledge, coaching and mentoring, opportunities to try out new methods and approaches, collegial support groups, and/or lesson study similar to the Japanese model cited in the research.

Since McGraw-Hill's *Math Connects* series provides a comprehensive, coherent curriculum, a rich variety of corresponding assessment measures, and related professional development features, using the program as the basis for the district's professional development offering will ensure that staff development, curriculum, and student assessment measures are aligned, as recommended by research. Over the long term, as the research suggests, such a program of professional development, rich in mathematical content knowledge and important mathematical ideas integrated in a logical sequence of lessons, is likely to improve teacher practice—with the ultimate goal of improving students' mathematical proficiency.

Technology

Pogrow (2004) claims to have developed a math instructional program that increases basic skills, enhances problem solving and increases student interest in math. In addition to using technology to grab student interest and to some degree automate instruction, Progrow's method attempts to build new approaches to teaching through simulations and exploratory learning. While no data is presented, Pogrow suggests that his approach produces outcomes that lead to gains in achievement as well as comprehension skills and self-concept.

Ertmer (2005) notes that while all conditions for high level of technology integration in the classroom are present, technology use is lower than expected. Ertmer points to ready access to technology, appropriate training for teachers and an environment that is technology friendly. However, she notes that barriers still exist. One specific barrier identified, specifically in math, reading and science is the pedagogical beliefs of teachers. The article lays out a case and components for further research on teacher beliefs as related to technology integration.

Several articles note the use of technology as important to bring application more closely to school curriculum. Ross (2004) highlights a locally developed curriculum project called the Math-Science Investigation (MSI). The article outlines technology and other reforms implemented with community participation. The reform integrates a more application based approach to math and science instruction.

In a related approach, Chen, Benton, Cicatelli and Yee (2004), present a generic case for collaboratives among teachers, schools communities and professionals to enhance learning. Again, through technology, a more hands on approach with an applied curriculum is the key.

How McGraw-Hill's Math Connects Series Relates to Scientifically Based Research on K-Algebra Technology and Mathematics Instruction

McGraw-Hill's *Math Connects* series provides numerous adjunct materials designed to technologically complement student learning and assignments. For example, all chapters of the Teacher's Edition of the Algebra book provide a section titled "Technology Solutions". A matrix is provided referencing the trademarked TeacherWorks CD-ROM. This CD addresses some of the key components of best practice referenced in the research literature. Units are provided for exploratory learning and an application based activity, noting activities for below grade level students, on-level students, above grade level students and/or English language learners. As well, a professional development section is included. Also available is a student CD-ROM with daily assignments, student worksheets and audio and a DVD showing Math in applied situations is available.

The same technology applications are available in each chapter of the grade 7 text.

Reading in the Content Areas

An argument can be made that good practices in the teaching of reading can be applied to the teaching of math (Center for the Education and Study of Diverse Populations, 2006). Making the comparison to reading, the article notes:

- mathematics should be enjoyable, limited to the real world with a focus on ideas
- research supports explicit instruction and applied problem solving
- students need concrete as well as abstract learning experiences
- starting with number sense can lead to presentation of big ideas
- students need to communicate their math ideas as a component of understanding
- questioning is relevant in the revealing of a student's math thinking
- fluency and flexibility with math facts and figures leads to conceptual understanding
- connections between new material and prior knowledge is important
- a variety of strategies are needed for computation and problem solving
- problem solving should be relevant to the students life and interests
- vocabulary in math needs to be in a conceptual context
- reflecting on patterns and applications are critical
- checklists and rubrics are useful in assessment

Hall (2004) adds that the teaching of math often involves reading. Hall reviews the research with respect to teacher's beliefs about the teaching of reading within a content area. The results of the study suggest that the importance of teaching reading in the content area needs to be stressed during preservice teacher training, then followed with emphasis during the inservice training years.

How McGraw-Hill's Math Connects Series Relates to Scientifically Based Research on K-Algebra Reading in the Content Areas

Opportunities to provide some reading instruction are included in all teacher editions at all levels of McGraw-Hill's *Math Connects* series. Consistent with the literature, all chapters at all levels attempt to connect number sense to Big Ideas (a section in all teacher editions). As well, all grade level materials provide a section of co-curricular links.

For example, at grade K, chapter 10, five learning stations are presented: Reading, Health, Art, Science and Social Studies. For the Reading station, there is an activity regarding listening to a story (The Shape of Things), drawing pictures of objects in the story and teacher notes suggesting reading the story aloud.

At grade 2, chapter 5 provides the same 5 learning stations. The Reading station presents an exercise reading sentences from a story given. Since the sentences are not in order, a problem solving exercise (see literature review above) of ordering the sentences according to the story is presented.

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